Joint Dynamics of Implied Volatility of Liquid DAX Components

A Master Thesis Presented

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Declaration of Authorship

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Berlin, September 02, 2009

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Abstract

Implied volatility can be considered as a function of strike level and time to maturity. As it is calculated from the actual trading options, it contains dynamic, multi-dimensional information of options, modelling the implied volatility is an interesting task for researchers. Dynamic semiparametric factor models (DSFM) are used to model the implied volatility surface. It employs semiparametric factor functions and time variate loadings to reduce the dimensions of the data. This master thesis applies joint analysis with the time variate factor loadings resulted from DSFM, in order to discuss the relationship between index options and stock options. The data of DAX index option and its liquid components stock options will be applied in analysis. The result of the joint analysis shows, that the index option has long term relationship with its stock options. It is unlikely to disperse the risk by trading the stock options under the same index.

Keywords: implied volatility surface, dynamic semiparametric factor models, option pricing model, joint analysis

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1 Introduction

The Black-Scholes model is one of the most used option pricing model in financial theory. Implied volatility (IV) is one of the most important parameters in financial practice. In practice, the traders use the implied volatility as an approximation of the option price. As the elements except implied volatility in Black-Scholes formula can be observed from the financial market, implied volatilities are derived from the Black-Scholes formula by plugging the market values of the parameters. The implied volatilities are modeled as a three dimensional surface together with time to maturity τ and the strike level, moneyness κ. Since there are a lot of trades with different time to maturities and strike prices on every trading day, which correspond to different option prices, the implied volatility is a dynamic, multi-dimensional concept. According to the call and put parity, the option prices are equivalent, no matter they are call or put options, if the other parameters determining the option prices are identical.

A lot of research work has been done in modelling the implied volatility surface. In this paper, the dynamic semiparametric factor models (DSFM), which have been well described by Fengler et al. (2007) and updated in Park et al. (2009), are applied to model the EUREX options data. The DSFM are built up as a combinition of the smoothing functions and a random loading vectors. As it is displayed in the equation:

$$Y_{t,j} = m_0(X_{t,j}) + \sum_{l=1}^{L} Z_{t,l} m_l(X_{t,j}) + \epsilon_{t,j}$$

m_l is the unknown nonparametric function, which is time invariant and $Z_{t,l}$ is the loading for factor m_l. Both of them are simutanously estimated from the data.

To study the characteristics of the implied volatility, the DAX traded at EUREX, the S&P 500 at NASDAQ and the FTSE at London Stock Exchange are often mentioned index options. For example, Fengler et al. (2007) used the DAX index options. In Cont and Fonseca (2002), both the S&P 500 and FTSE index options were taken in analysis. Few papers discussed the modelling of the stock options. However, in financial market, the trading sizes of the stock options are getting larger and larger. In recent years, the trading volumes on Eurex are increasing quite fast. Total trading volumes in the first four months of 2007 reached 618 million contracts. In April, equity derivatives reached a total of 53 million contracts, which is an increase of approximately

50 percent. Because of this fast development in financial derivatives market, the relationship between the index and stock options are also interesting for researchers. Are the stock options cointegrated with the index options? Do they have the same tendence over the time? Can the investors hedge the risk by invest in another style of the options? Those are the motivations of this thesis to use the data of stock options from DAX 30 trading at EUREX to compare them with the DAX index options.

In modelling the implied volatility surface, Cont et al. (2002) modelled the implied volatility in stochastic methods. Hafner (2004) applied factor analysis. Dumas et al. (1998) used smoothing methods to model the implied volatility surface. Fengler et al. (2007) and Park et al. (2009) modified the dynamic semiparametric factor models (DSFM), using the nonparametric smoothing function and factor analysis to reduce the dimensions of the implied volatility. As the factor loadings of DSFM simplifies the dynamic analysis, this thesis will apply the DSFM to model the implied volatility surface.

Cont and Fonseca (2002) analysed seperately the S&P 500 and FTSE options with principal components analysis. Christensen and Nielsen (2002) tested the stationary and the cointegration of implied volatilities in S&P500. Cao et al. (2009) tested cointegration as well as the impulse responses of the two index options. This master thesis will test the stationary and the cointegration of the factor loadings from the result of DSFM to reveal the mutual relationship between DAX index options and its liquid components of stock options.

The thesis is organized as following: The second part will make a short review of the Black-Scholes formula for further calculating of implied volatilites. The third part will introduce the methods of the dynamic semiparametric factor models (DSFM). The fourth part will explain the data sources and the selection of the data set. Then the data will be applied into the DSFM. The fifth part will take advantage of the results from the fourth part to do the joint dynamic analysis between the DAX and the liquid components. The last part will give a short conclusion and discuss the left questions.

The calculation and data analysis for this paper are done with the softwares of R, Matlab and EViews.

2 The Implied Volatility

2.1 The Concepts of the Implied Volatility

As the Black-Scholes option pricing formula was proposed in 1973, it is under the assumptions that the underlying asset follows a geometric Brownian motion in continuous time and the stock pays no dividends , which is described as:

$$dS_t = \mu \cdot S_t dt + \sigma \cdot S_t \cdot dW_t, \tag{2.1}$$

W_t is Wiener process. (see Franke et al. (2008) P67)

The complete assumptions were given as follows:

- The short-term interest rate is known and is constant through time
- The stock price follows a random walk in continuous time. The variance rate of the return on the stock is constant.
- The stock pays no dividends or other distributions.
- The option is "European", that is, it can only be exercised at maturity.
- There are no transaction costs in buying or selling the stock or the option.
- It is possible to borrow any fraction of the price of a security to buy it or to hold it, at the short-term interest rate.
- There are no penalties to short selling.

But in the real market, it is hard to find this perfect market conditions. According to Merton (1973), Black-Scholes formula is still valid when

- The interest rate is stochastic.
- The stock pays dividends.
- The option is exercisable prior to expiration.

Moreover, it is shown that as long as the stock price dynamics can be described by a continuous-time diffusion process whose sample path is continuous with probability one, then their arbitrage technique is still valid.

In conclusion, the Black-Scholes model can not only be applied under the perfect market assumptions but also matches the activities on the real market. As the assumptions of the Black-Scholes model are developed by Merton (1973), the implied volatility of the European as well as the Amrican style options can both be derived by applying the BS formula:

$$C(S,\tau) = e^{(b-r)\tau} S\Phi(y+\sigma\sqrt{\tau}) - e^{-rt} K\Phi(y) \tag{2.2}$$

where

$$y = \frac{ln\frac{S}{K} + (b - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} \tag{2.3}$$

Φ denotes the standard normal distribution:

$$\Phi(y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y} e^{-\frac{z^2}{2}} dz \tag{2.4}$$

(see Franke et al. (2008) P82)

b denotes the cost of carry, which is defined as the cost for paying dividends (d):

$$b =: r - d \tag{2.5}$$

In the BS formula, the value of the option prices, the time to maturiy (τ), the strike price (K), spot price (S_t), the risk-free rate (r) and the dividend (d) can all be observed from the actual trading market. From the put - call parity, we know that under the same conditions, the option prices should be equivalent whether the options are puts or calls.

2.2 Calculation of the Implied Volatility

The calculation of the implied volatility is under the assumption that the Black-Scholes pricing formula reflects the real market options prices. It differs from the historical volatility, which is the second moment of the historical market prices. The implied volatility is derived from the option pricing model, as we applied here the Black-Scholes pricing formula. In financial derivatives analysis, implied volatility is a criterion of the options pricing process, because it reflects the pricing information on the market. Implied volatility integrates the market values of the parameters, which appear in the pricing formula, such as time to maturity, risk-free rate, dividend, strike price, spot price and the futures price.

Thus, the implied volatility is calculated by solving the equation of

$$C_t^{BS} S_t, r_t, K, \tau, \sigma_t(K,\tau) - \widetilde{C_t} = 0, \tag{2.6}$$

Here we define the strike level $\kappa := \frac{K}{F_t}$ as moneyness. F_t is the futures price. In our case, the futures price F_t is derived from the spot price S_t, which would be calculated as $F_t := S_t e^{b\tau} = S_t e^{(r-d)\tau}$.

Thus, implied volatility is determined by three elements, the time to maturiry (τ), the moneyness (κ) and the market options prices. As implied volatility is an important element to determining the options price, the plotting of the implied volatility is then a three dimensional analysis of the relations among the implied volatility (σ_I), time to maturity (τ) and the moneyness (κ).

In Figure 1 and Figure 2, two examples of the results of our calculation have been taken to show the general relationship among the 3 variables.

Figure 1 displays the 3-dimentional plotting of the implied volatilites on 15th Oct. 2007, which are calculated from ODAX data and Daimler Chrysler options data. It shows that at the same moneyness, the implied volatility is getting higher, while the time to maturity is getting shorter. Meanwhile, at the same time to maturity, the implied volatility is decreasing, while moneyness is increasing. Both Daimler Chrysler's and DAX's options have the same relationship among these three variables, since they are calculated from the same model. The general values of implied volatilities of ODAX on this day is smaller than that of Daimler Chrysler stock options.

Figure 2 shows the distribution of the moneyness at different time to maturity on that day. In both stock and index options, most options are traded at the time to maturity from 0.02 to 0.5. The plotting of DAX index options is denser than the Daimler Chrysler stock options, since the observations of DAX options are much more than Daimler Chrysler options, which means, the tradings on DAX index options are much more active and frequent than Daimler Chrysler stock options.

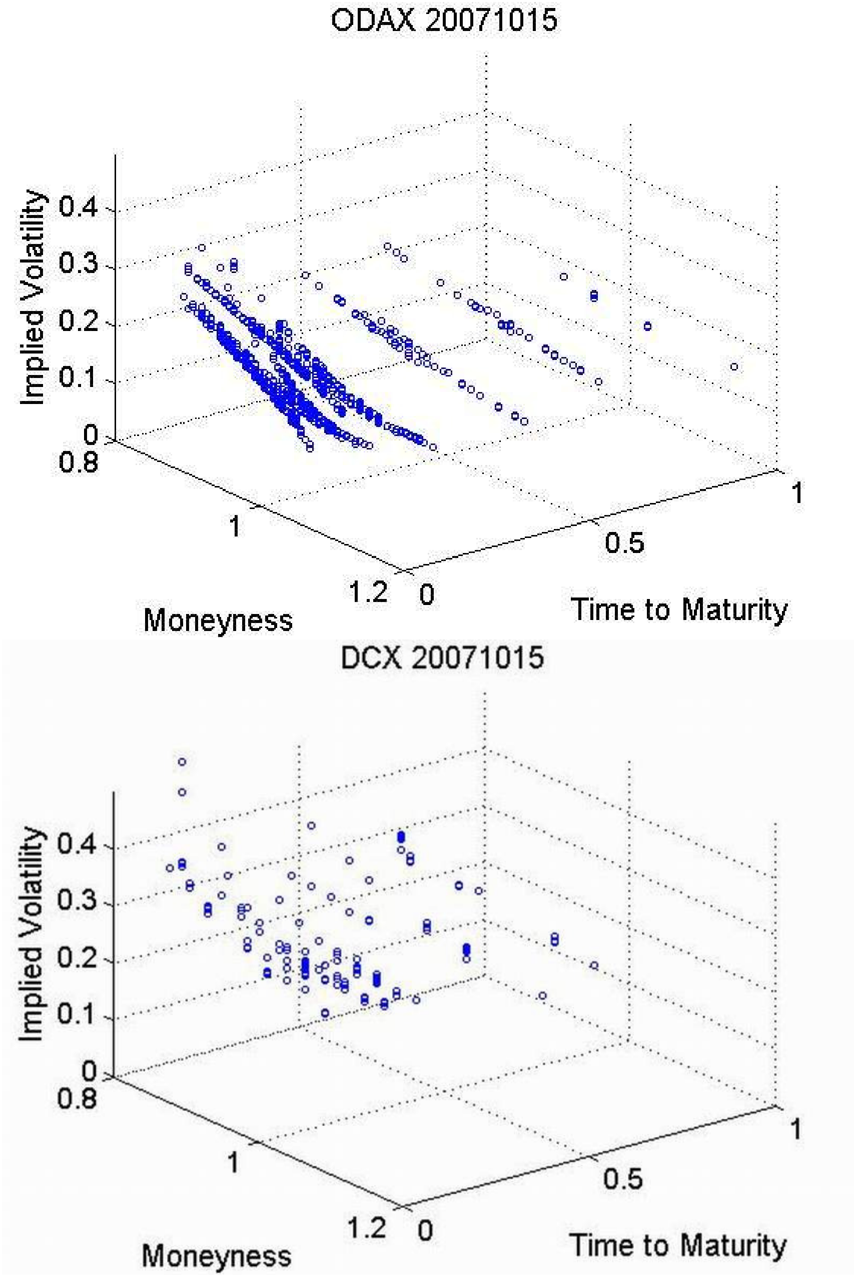

Figure 1: Scatterplot of implied volatilities of ODAX and Daimler Chrysler options on 15th Oct. 2007

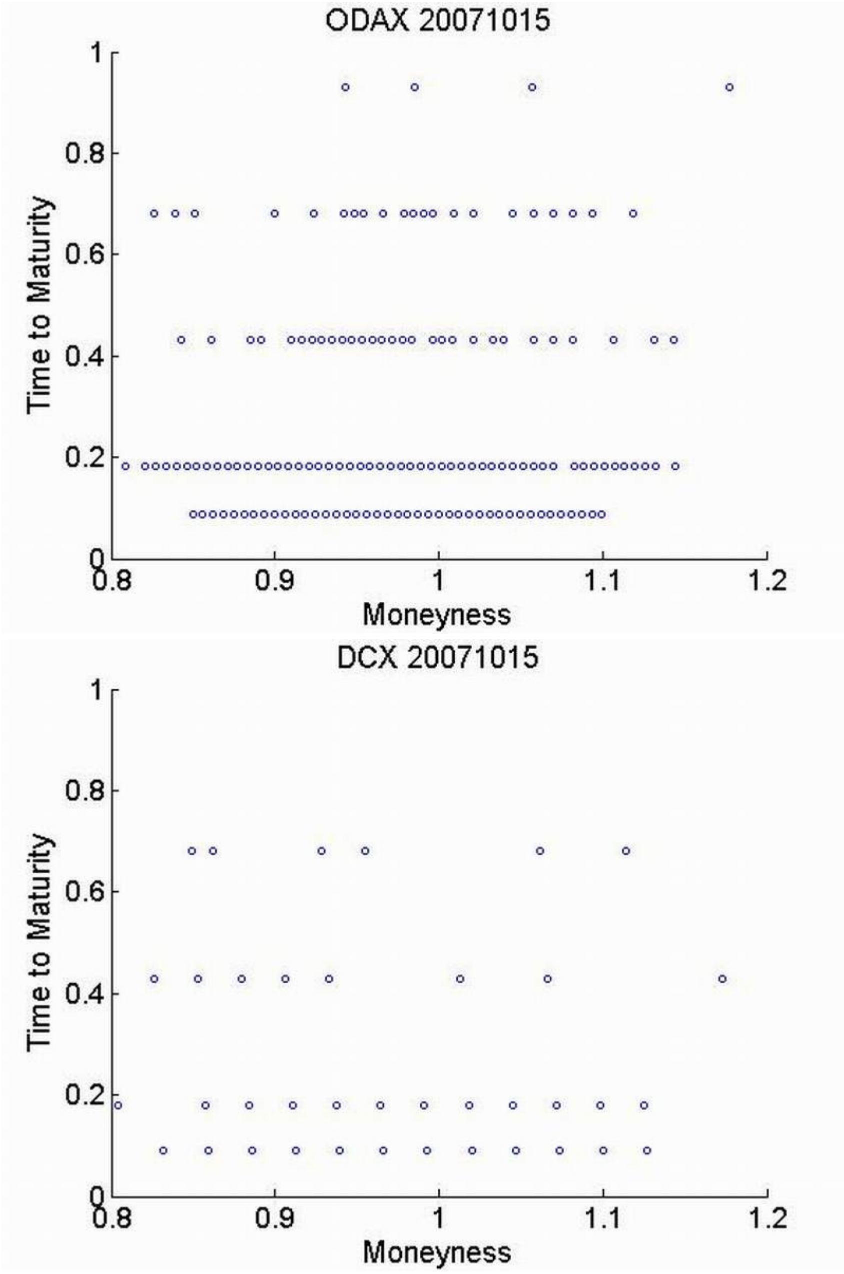

Figure 2: Plot of moneyness and time to maturity of ODAX and Daimler Chrysler options on 15th Oct. 2007

3 Dynamic Semiparametric Factor Models (DSFM)

The scatterplotting of the implied volatility as showed in the last section is a discrete time plotting for every deal on each trading day. Then, the implied volatility surface (IVS) can be considered as a continuous surface by modelling $\sigma \rightarrow \sigma_I(\kappa, \tau)$.

The DSFM has a nonparametric part to avoid the mismodelling of the data, combined with the factor analysis to reduce the dimensions and explore the moving tendence through the period. The model was well defined in Fengler et al. (2007), as well as in the papaer of Park et al. (2009).

Consider $Y_t = (Y_{t,1}, ..., Y_{t,J})^T$ as an orthogonal L-factor model an observable J-dimensional random vector, which can be represented as:

$$Y_{t,j} = m_{0,j} + Z_{t,1} m_{1,j} + ... + Z_{t,L} m_{L,j} + \epsilon_{t,j} \tag{3.1}$$

where $m_{l,j}$ are common factors, $\epsilon_{t,j}$ is error term or specific factor and the coefficients $Z_{t,l}$. For the index t=1,...,T represents the time evolution of the whole system, Y_t can be considered as a multi-dimensional time series.

In application, there will be normally explanatory variables $X_{t,j} \in \mathbb{R}^d$, that will influence the nonparametric factor functions $m_{L,j}$. So the model (3.1) will be generalized as:

$$Y_{t,j} = m_0(X_{t,j}) + \sum_{l=1}^{L} Z_{t,l} m_l(X_{t,j}) + \epsilon_{t,j}, 1 \leq j \leq J_t \tag{3.2}$$

In our case,

- Y_t is defined as logarithmised implied volatility: $Y_t := log(\sigma_I(\kappa, \tau))$.
- t is the index of day (t=1,...,T).
- j is the index of options on day j (j=1,...,J), that means, the options are intraday values, there are many deals during one day.
- $m_{l,j}$ (l=0,...,L) is time invariant factor functions
- $X_{t,j} := (\kappa_{jt}, \tau_{jt})$, which is a 2-dimentional vector.
- $Z_{t,l}$ is loading for factor $m_{l,j}$, which variate with day t.

- $\epsilon_{t,j}$ has zero mean and finite second moments, which is conditioned on $X_{t,j}$ and independent from $Z_{t,l}$.

3.1 Estimation of the DSFM

The detailed estimation process is referred to Park et al. (2009) and Giacomini et al. (2008).

To estimate $\hat{m}_l$, we define the least squares estimators $\hat{Z}_{t,l}$ (l=1,...,L,t=1,...,T) and $\hat{A}$=($\hat{\alpha}_{l,k}$):

$$S(A,z) \equiv \sum_{t=1}^{T}\sum_{j=1}^{J}\left\{Y_{t,j} - (1, z_t^T)A\psi(X_{t,j})\right\}^2 = min_{A,z}! \tag{3.3}$$

where:

- z= $(z_1^T, ..., z_T^T)^T$ for L-dimensional vectors z_t,
- functions ψ_1,...,ψ_K:$[0.1]^d \to \mathbb{R}$ can be normalized so that $\int_{[0,1]^d} \psi_k^2(x)dx = 1$, with $K \geq 1$,
- A=($\alpha_{l,k}$) is an $(L+1) \times K$ and $\psi = (\psi_1, ..., \psi_K)^T$

To find a solution of ($\hat{A}$,$\hat{Z}$) of the minimization problem (3.3), The paper by Park et al. (2009) proposes a Newton-Raphson algorithem.

Given (α^{OLD},Z^{OLD}), the Newton-Raphson algorithm updates the equation for (α^{NEW},Z^{NEW}):

$$\begin{pmatrix}\alpha^{NEW}\\ Z^{NEW}\end{pmatrix} = \begin{pmatrix}\alpha^{OLD}\\ Z^{OLD}\end{pmatrix} - F_*'(\alpha^{OLD}, Z^{OLD})^{-1}F(\alpha^{OLD}, Z^{OLD}) \tag{3.4}$$

where $F_*'(\alpha, z)$ is the restriction to F_* of the linear map defined by the matrix $F'(\alpha, z)$. F_* is the linear space of values of (α,z).

It is argued, that the equation (3.4) converges to a solution of (3.3) under some weak conditions on the initial choices of ($\alpha^{(0)}$,$Z^{(0)}$), which satisfy the conditions that (i) $\sum_{t=1}^{T} Z_t^{(0)} = 0$. The marix $\sum_{t=1}^{T} Z_t^{(0)} Z_t^{(0)T}$ and the map $F_*'(\alpha^{(0)}, Z^{(0)})$ are invertible. (ii) There exists a version ($\hat{\alpha}$,$\hat{Z}$) with $\sum_{t=1}^{T}\hat{Z}_t = 0$

such that $\sum_{t=1}^{T} \hat{Z}_t \hat{Z}_t^{(0)}$ is invertible. $\hat{\alpha}_l = (\hat{\alpha}_{l1}, ..., \hat{\alpha}_{lK})^T$ for l=0,...,L are linearly independent. So that one has:

$$\hat{Z}_t^T \hat{A} = \hat{\alpha}_0^T + \sum_{l=1}^{L} \hat{Z}_{t,l} \hat{\alpha}_l^T = (\hat{\alpha}_0^T + \sum_{l=1}^{L} \overline{\hat{Z}_l} \hat{\alpha}_l^T) + \sum_{l=1}^{L} (\hat{Z}_{t,l} - \overline{\hat{Z}_l}) \hat{\alpha}_l^T \overset{let}{=} \hat{\alpha}_0^{*T} + \sum_{l=1}^{L} \hat{Z}_{t,l}^* \hat{\alpha}_l^T = \hat{Z}_t^{*T} \hat{A}^*$$

where $\overline{\hat{Z}_t} = T^{-1} \sum_{t=1}^{T} \hat{Z}_{t,l}$, $\hat{Z}_t^{*T} = (1, \hat{Z}_t^{*T})$ and $\hat{A}^*$ is the matrix obtained from $\hat{A}$ by replacing the its first row by $\hat{\alpha}_0^{*T}$. The minimization problem (3.3) has no unique solution.

Define $\widetilde{B} = \begin{pmatrix} 1 & 0 \\ 0 & B \end{pmatrix}$. If $(\hat{Z}_t, \hat{A})$ or $(\hat{Z}_t, \hat{m} = \hat{A}\Psi)$ is a minimizer, the $(B^T \hat{Z}_t, \widetilde{B}^{-1} \hat{m})$ is also a minimizer, where B is an arbitrary invertible matrix. $\widetilde{B}$ assures that the first component of $\widetilde{B}^T \hat{Z}_t$ equals 1. In particular, with the choice B = $(\sum_{t=1}^{T} \hat{Z}_t^{(0)} \hat{Z}_t^T)^{-1} \sum_{t=1}^{T} \hat{Z}_t^{(0)} \hat{Z}_t^{(0)T}$, we will get for $\hat{Z}_t^* = B^T \hat{Z}_t$ with $\sum_{t=1}^{T} \hat{Z}_t^{(0)} (\hat{Z}_t^* - Z_t^{(0)})^T = 0$ Note the $\hat{m} = \hat{A}\Psi$ is chosen such that the $\hat{m}_1$,...,$\hat{m}_L$ are orthonormal in $L_2[0,1]^d$, then the matrix B would be an orthogonal matrix and the underlying time series Z_t is estimated up to such transformations.

3.2 The choice of the model size

The choice of the model size L is defined as computing the residual sum of squares

$$RV(L) = \frac{\sum_t^T \sum_j^{J_t} Y_{t,j} - \hat{m}_0(X_{t,j}) - \sum_{l=1}^{L} \hat{Z_{t,l}} \hat{m}_l(X_{t,j})^2}{\sum_t^T \sum_j^{J_t} (Y_{t,j} - \bar{Y})^2} \tag{3.5}$$

1-RV(L) is defined as the explained variance. For better fit of the model, the L should be chosen so that the explained variance should be as high as possible.

4 Data Descriptioin and Model Fitting

4.1 Data

The options data used in this thesis are from the database of the Collaborative Research Center 649. The database contains the actual trading information of EUREX, which is one of the largest financial derivatives exchange market in Europe. We use the options trading at the EUREX in Frankfurt (Main) during the whole year 2007, from January 2007 to December 2007 (251 trading days). The DAX index option (ODAX) is European style, that is, an option can only be exercised on the last trading day of the option. The stock options on German shares (OSTK) are different from ODAX. Generally, an OSTK option contract gives the buyer the right to buy or sell (put or call option) 100 shares of the underlying security. Allianz and Münchener Rückversicherung have a contract size of 50 shares. The OSTK are American style, that means, the stock options can be exercised before expiration day. Besides, both ODAX and OSTK are cash settled and have a minimum price movement of 0.01 EUR. The expiration day is on the third Friday of every month. The expires of ODAX are the three nearest calendar months, the three following months of the cycle March, June, September, and December and the two following months of the cycle from June to December. Meanwhile, the OSTK can be divided into 3 groups according to their expiration months, which is summarized in Table 1, the companies with "*" are on the DAX 30 list in 2007. It shows that each stock option has different expiration design. (quoted from www.eurexexchange.com)

All stock options on DAX components are available for the three nearest contract months, for 6, 9 and 12 months. Group B and C are also listed for longer maturities of up to 24 and 60 months.

Thus, the buyers have broader range of stock options to choose. Stock options have their special characteristics, so that the institutional and private investors can offset price risks in their equity positions and to profit from both upward and downward price movements of individual shares. The stock options on DAX components provide a multitude of investment strategies to hedge the market risk on the German equity market. (Trading strategies can also be referred to broshure from EUREX: "Equity and Equity Index Derivatives Trading Strategies") Because of the liquidity and the risk-hedging characteristics, the number of traded contracts in DAX components' stock options reached more than 63.6 million contracts in 2002.

Group A	Group B	Group C
1, 2, 3, 6, 9 and 12 months	1, 2, 3, 6, 9, 12, 18 and 24 months	1, 2, 3, 6, 9, 12,18, 24, 30, 36, 48 and 60 months
Adidas (ADS)* Continental (CON)* Degussa (DGX) Deutsche Börse (DB1)* Deutsche Post (DPW)* Dresdner Bank (DRB)* EPCOS (EPC) Fresenius Medical Care (FME)* Henkel Vz. (HEN3)* Karstadt (KAR) Linde (LIN)* MAN (MAN)* Metro (MEO)* MLP (MLP) Porsche (POR3) Preussag (PRS) RWE (RWE)* Schering (SCH) Thyssen Krupp (TKA)*	BASF (BAS)* E.On (EOA)* Lufthansa (LHA)* Münchner Rückversicherung (MUV2)* Bay. Hypo. Und Vereinsbank (HVM) VW (VOW)* Siemens (SIE)*	Allianz-Holding (ALV)* Deutsche Bank (DBK)* Daimler Chrysler (DCX)* Deutsche Telekom (DTE)* Infineon (IFX)* SAP (SAP)*

Table 1: EUREX OSTK groups according to expirations

Since we argued in the first part, that the Black-Scholes formula is also valid, evenwhen an options can be exercised before expiration, the Black-Scholes formula is still applied here to calculate the implied volatility for both styles of the options. However the limitation of accuracy for using the Black-Scholes formula in American style options should be considered, for we can only assume that an American style option also exercise at the expiration. The earlier exercise won't be taken into account, since it is hard to summarize the real data from the market and the problem will be too complicated.

From the data set of EUREX, the following information has been selected for our analysis:

- 1) The dates of trading
- 2) The expiration dates with months and years: since the date of expiration on EUREX is the third Friday in every month, they only recorded the expiration years and months. The time to maturities are then calculated

from the differences of the exact expiration dates and the trading dates. As this calculation is included the weekends, the annualized time to maturity is then τ=T/365.

- 3) The type of the options: put or call
- 4) The strike price (K)
- 5) The options price

The above are all the intraday data for both ODAX and OSTK. In calculating, the DAX index is a capital-weighted performance index, that is, dividends less than corporate tax are reinvested into the index. (Deutsche Börse (2009)) Therefore, dividend payments have no impact on index options. For the EUREX liquid components, 7 companies' options are chosen, which are on the list of DAX 30 and have the largest trading sizes and trading frequencies.

These 7 companies are:

- 1) Daimler Chrysler
- 2) Siemens
- 3) Allianz
- 4) Deutsche Bank
- 5) SAP
- 6) Muenchener Rück
- 7) Deutsche Telekom

The main task of this thesis is to give a brief perspective of the relationship between the index options and stock options. Thus, we only calculated the implied volatility of the options trading in 2007 for ODAX and the 7 most traded liquid components. For dividends paying is an important feature of stocks, the dividends of the equities are also considered in calculating the implied volatilities for the stock options.

The spot prices (S_t) and the annually dividend rates (d) for each company are taken from Thomson Financial Datastream. As it is really hard to get the intraday spot prices, instead, the closing prices on each trading day are used in calculation. The futures prices (F_t) are not available, either. Thus, they were calculated according to the equation: F_t=$S_t e^{b\tau} = S_t e^{(r-d)\tau}$.

The Euribor (Euro Interbank Offered Rate) is the rate at which euro interbank term deposits are being offered by one prime bank to another within the EMU zone. In our case, the daily Euribor is chosen as daily risk-free interest rate, since it reflects the markt situation in Europe and our data are selected from the German market. The daily Euribor has different values according to different time to maturities. The everyday's rates vary from 1 week to 12 months. Figure 3 shows the Euribors in March 2007 as an obvious example. Every line represents one day. Since they are almost straight lines, the median value of every day's rates are applied as the daily risk-free rate in calculation.

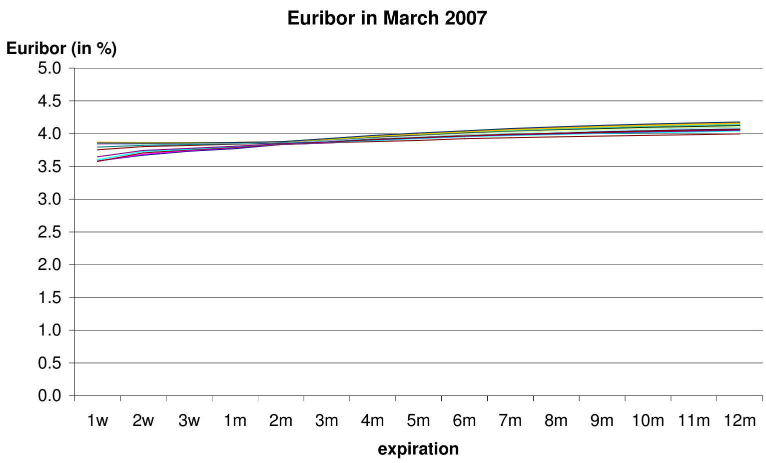

Figure 3: Euribor in March 2003

For the further model fitting, we restricted the time to maturity τ within the range of [0.02,1], the moneyness $\kappa = \frac{K}{F_t}$ is restricted in [0.8,1.2], which is around at-the-money. The calculated implied voaltility is restricted in the range of [0.02,1]. For there are few observations out of these restrictions. Furthermore, the data outside these ranges moved irregularly.

Table 2 shows the descriptive statistics of our data. Since the data set for ODAX is quite large, the repeated items are ignored. That means, the deals with the same time to maturity, same moneyness and implied volatility on the same day will only leave 1 observation in the data set. As the information shown in Table 3, generally, the liquid components have higher implied volatilities than ODAX. This matches the feature of the liquid components, that the liquid components cost higher risk, but will bring better profits because of its flexibility. The kutosis of the implied volatilities are relatively high, whereas the skewnesses are relatively low. Thus, the values of the implied volatilies are concentrated around mean and median values.

		Min.	Max.	Mean	Median	Stdd.	Skewn.	Kurt.
ODAX	time to maturity	0.019	1.000	0.145	0.085	0.164	2.478	9.475
	moneyness	0.800	1.200	0.984	0.991	0.057	-0.419	3.876
	implied volatility	0.002	0.999	0.199	0.191	0.061	1.922	16.607
Daimler Chrysler	time to maturity	0.019	1.000	0.199	0.120	0.210	1.797	5.612
	moneyness	0.800	1.200	0.994	0.996	0.078	-0.087	2.858
	implied volatility	0.027	0.999	0.316	0.304	0.077	1.249	7.968
Siemens	time to maturity	0.020	1.000	0.216	0.120	0.224	1.644	4.923
	moneyness	0.800	1.200	0.993	0.995	0.076	0.006	2.968
	implied volatility	0.037	0.996	0.282	0.275	0.071	1.343	9.974
Allianz	time to maturity	0.019	1.000	0.194	0.118	0.204	1.858	5.922
	moneyness	0.800	1.200	1.003	1.003	0.074	-0.023	3.073
	implied volatility	0.019	0.998	0.260	0.253	0.066	2.042	15.146
Deutsche Bank	time to maturity	0.019	1.000	0.204	0.121	0.214	1.780	5.535
	moneyness	0.800	1.200	1.004	1.003	0.074	0.014	3.036
	implied volatility	0.024	0.983	0.279	0.270	0.079	1.327	8.155
SAP	time to maturity	0.019	1.000	0.225	0.145	0.219	1.616	4.963
	moneyness	0.800	1.200	1.011	1.010	0.067	-0.027	3.323
	implied volatility	0.017	0.999	0.267	0.259	0.060	2.711	20.288
Münchener Rück	time to maturity	0.019	1.000	0.206	0.137	0.201	1.776	5.728
	moneyness	0.800	1.200	1.010	1.012	0.069	-0.172	3.503
	implied volatility	0.033	0.986	0.228	0.221	0.061	1.914	14.317
Deutsche Telekom	time to maturity	0.019	1.000	0.254	0.153	0.248	1.310	3.656
	moneyness	0.800	1.200	1.011	1.009	0.068	-0.037	3.594
	implied volatility	0.042	0.992	0.244	0.234	0.076	2.268	13.677

Table 2: Data: descriptive statistics

4.2 Fitting the Dynamic Semiparametric Factor Models

As explained in section 3, Y_t will be considered as the logrithem of implied volatility: Y_t:=log($\sigma_I(\kappa,\tau)$) .

In order to fit our data in DSFM, the choice of L should be measured at first to choose the proper explained variance. That is, to get a better (1-RV(L)).

options	L	1-RV(L)	L	1-RV(L)	ΔRV	L	1-RV(L)	ΔRV
ODAX		0.721		0.734	0.013		0.742	0.009
Daimler Chrysler		0.351		0.469	0.119		0.495	0.025
Siemens		0.518		0.557	0.040		0.583	0.026
Allianz		0.421		0.461	0.040		0.480	0.019
Deutsche Bank	1	0.540	2	0.587	0.048	3	0.604	0.017
SAP		0.380		0.421	0.041		0.444	0.023
Münchener Rück		0.438		0.488	0.050		0.520	0.033
Deutsche Telekom		0.289		0.344	0.055		0.371	0.027
ODAX				0.745	0.003		0.750	0.004
Daimler Chrysler				0.518	0.023		0.529	0.011
Siemens				0.595	0.012		0.601	0.007
Allianz				0.490	0.010		0.502	0.012
Deutsche Bank			4	0.623	0.019	5	0.632	0.009
SAP				0.458	0.014		0.471	0.012
Münchener Rück				0.533	0.013		0.551	0.017
Deutsche Telekom				0.392	0.021		0.410	0.017

Table 3: Explained variances

The result of different L choices is in Table 3. Since every iteration produces a different value for RV(L), the last iteration has the lowest value for RV(L), which improves the value of (1-RV(L)), we take the (1-RV(L)) value generated by last iteration as the explained variance. With our data, the explained variance is monoton increasing, when L is also increasing. But the explained variances for ODAX data are overall higher than the stock options data. This may be caused by the stock options' trading style and the data distribution.

The result of the calculation displays in Table 3, from L=3 on, the differences of the explained variances are getting smaller. For example, the Δ RV of ODAX for (1-RV(L=4))-(1-RV(L=3)) is 0.003, whereas (1-RV(L=2))-(1-RV(L=1)) is 0.013. From the overall result of our data, the explained variances are increasing more slowly when L≥ 3. The results for the stock options are similar. Therefore, we choose L=3 to explore the dynamic factors and the factor loadings of our data.

Plugging the calculated implied volatility into DSFM with L=3, the implied volatility surface is modelled. Again, take the DAX index options and Daimler Chrysler stock options as an example. The implied volatility surface modelled by DSFM is shown in Figure 4. Since the implied volatility surface is a dynamic process changing with time, we still take an example on the day 15th Oct. 2007 and limit the time to maturity from 0 to 0.5. Comparing Figure 1 and Figure 4, the shapes of the scatterplotting and the surface match together, where the implied volatilities of Daimler Chrysler is generally higher than that of ODAX on that day.

At the same time, the factors and the factor loadings are produced. Figure 5-7 are the results of the $\hat{m}$s for ODAX and Daimler Chrysler stock options. According to the definition of DSFM, the results are not definite. The following plotting of $\hat{m}$s are under the condition that $\hat{Z}_1$s are positive. Since the factor loading of $\hat{m}_0$ is constant, we don't display $\hat{m}_0$.

In Figure 5, both $\hat{m}_1$s are smoother. Both of them are slightly under zero. But the $\hat{m}_1$ of ODAX has a small upward turn at the time to maturity betwenn 0.2 and 0.4.

Figure 6 and 7 are results of $\hat{m}_2$ and $\hat{m}_3$ for both options. The $\hat{m}_2$ and $\hat{m}_3$ for Daimler Chrysler stock options are much more fluctuated. The movement of the values are not regular. But the $\hat{m}_2$ of ODAX is smoother with a similar shape as its $\hat{m}_1$ plotting. The $\hat{m}_3$ of ODAX fluctuated at both small and large end of the time to maturity. In both $\hat{m}_2$ and $\hat{m}_3$ plotting, the Daimler Chrysler stock option is more random and the ODAX is stabler.

As the $\hat{m}$s are time invariate factor functions and the dimensions haven't been reduced in the $\hat{m}$s' plotting, the $\hat{Z}$s have to be considered here, since they are the factor loadings of $\hat{m}$s, which reduce the dimensions into the time depending series.

Figure 8-10 give a broad perspect on the time series of the $\hat{Z}$s for ODAX and 7 stock options, which follow the time periods through the 251 trading days. As discussed, $\hat{Z}_0$ is a constant. It isn't shown in the figure. The values of $\hat{Z}_1$s are between 0.5 and 1.2. And the values of $\hat{Z}_1$ for ODAX are generally higher than 7 stock options, which are displayed by the highest blue line in Figure 8. $\hat{Z}_2$ and $\hat{Z}_3$ move irregularly around 0. Some of them are highly positive, some are highly negative, that may be caused, because the signs of $\hat{Z}_2$ and $\hat{Z}_3$ are not

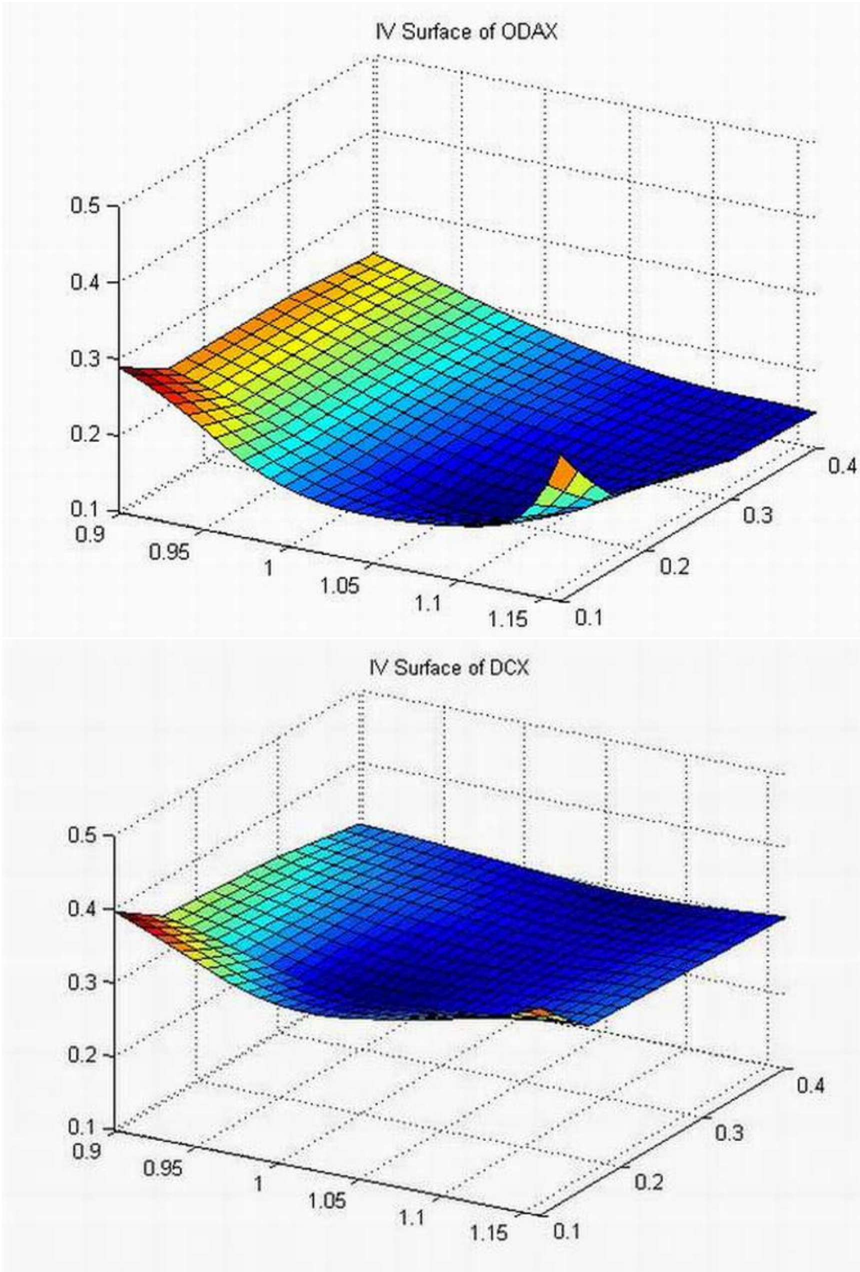

Figure 4: Implied volatility surfaces for ODAX and DCX on 15th Oct. 2007

fixed. Anyhow, around May and June 2007, there is a sudden fluctuate in both $\hat{Z}_2$ and $\hat{Z}_3$. There may exist some causes from the relevant companies.

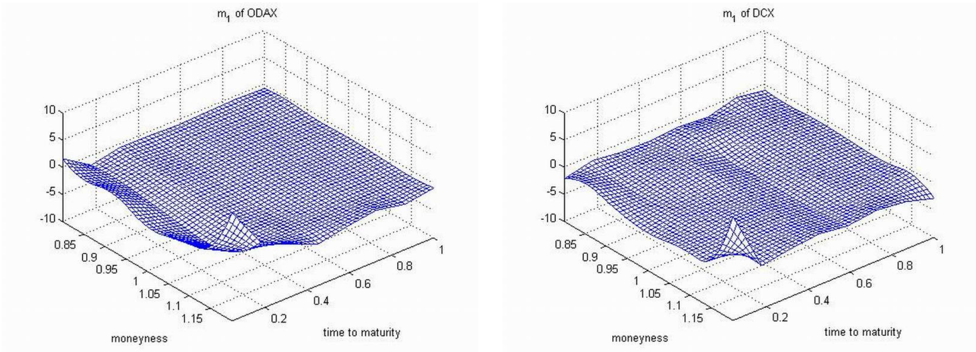

Figure 5: $\hat{m}_1$ of ODAX and DCX

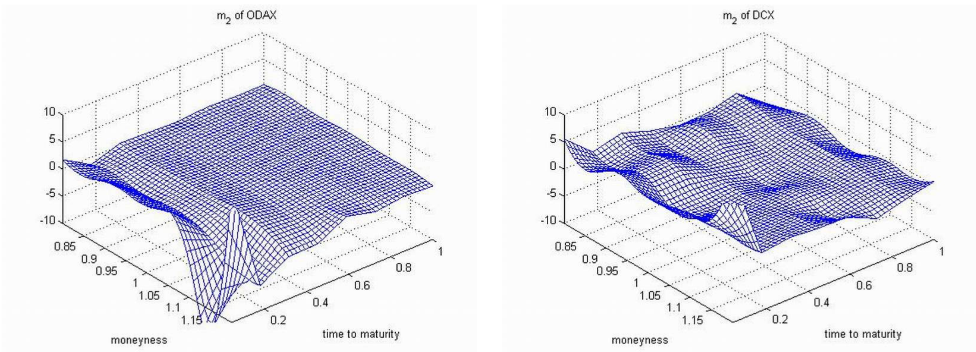

Figure 6: $\hat{m}_2$ of ODAX and DCX

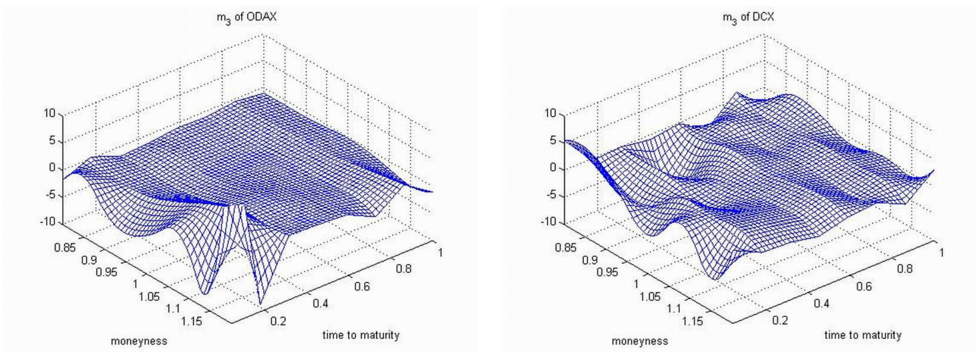

Figure 7: $\hat{m}_3$ of ODAX and DCX

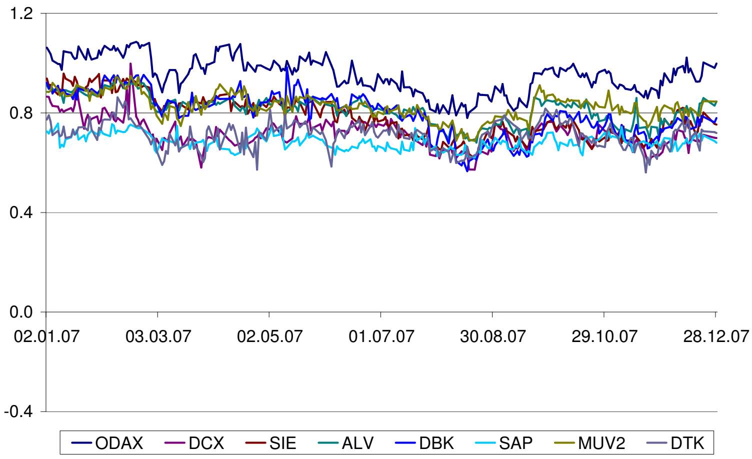

Figure 8: $\hat{Z}_1$ of ODAX and 7 liquid components

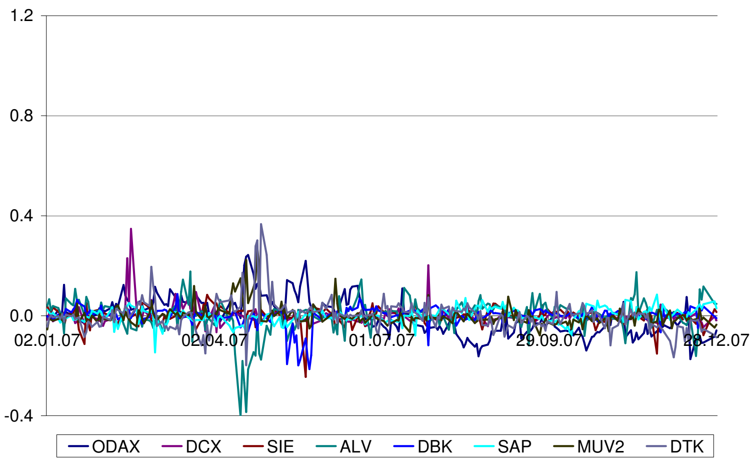

Figure 9: $\hat{Z}_2$ of ODAX and 7 liquid components

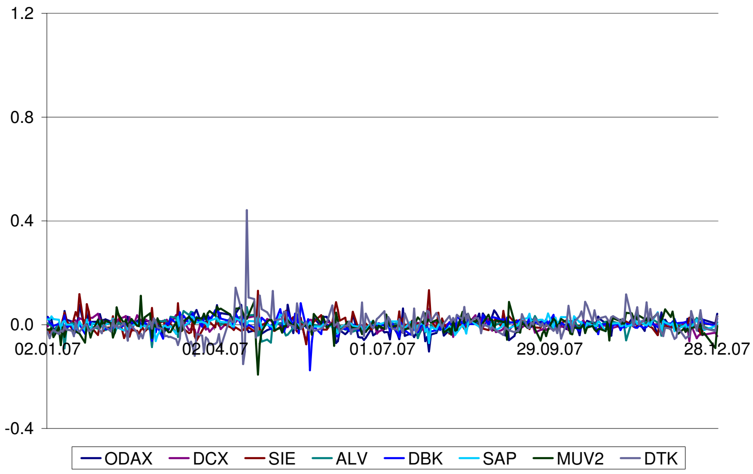

Figure 10: $\hat{Z}_3$ of ODAX and 7 liquid components

5 Joint Dynamic Analysis of the DAX and Liquid DAX Components

As the $\hat{Z}_1$s are the factor loadings of $\hat{m}_1$s, wheareas $\hat{m}_1$ reflects the tendence of the real data, we explore the relationships between $\hat{Z}_1$s instead of exploring the raw data. $\hat{Z}_1$ is a time series containing the information from Janurary to December 2007, it can be modelled as a verctor autoregression process, VAR(p):

$$\hat{Z}_t = \nu + \sum_{p=1}^{P} \Phi_p \hat{Z}_{t-p} + \mu_t \tag{5.1}$$

where $\nu = (\nu_1, ..., \nu_k)^T$ is a $k \times 1$ vector of intercept terms, Φ_p are $k \times k$ parameter matrices and μ_t is a k-dimensional white noise or innovation process that is $E(\mu_t)$=0, $E(\mu_t \mu_t') = \Sigma_\mu$ and $E(\mu_t \mu_s') = 0$ for $s \neq t$. The covariance matrix Σ_μ is assumed to be nonsigular. (referring to Lütkepohl (2007) and Cao et al. (2009))

Table 4 is the descriptive statistics of the analysed $\hat{Z}_1$. For all of the components, the means and medians are very close to each other and the standard deviations are relatively low. That means, there are no sudden turnovers in $\hat{Z}_1$ series.

	options	Min.	Max.	Mean	Median	Stdd.	Skewn.	Kurt.
	ODAX	0.780	1.085	0.950	0.958	0.071	-0.190	-0.633
	Daimler Chrysler	0.572	0.998	0.716	0.712	0.059	0.697	2.009
	Siemens	0.618	0.958	0.783	0.772	0.084	0.225	-0.956
$\hat{Z}_1$	Allianz	0.681	0.948	0.813	0.821	0.057	-0.194	-0.453
	Deutsche Bank	0.566	0.983	0.797	0.801	0.087	-0.334	-0.466
	SAP	0.620	0.757	0.683	0.683	0.030	0.249	-0.528
	Münchener Rück	0.688	0.949	0.825	0.826	0.052	-0.168	-0.145
	Deutsche Telekom	0.561	0.863	0.712	0.720	0.054	-0.268	-0.073

Table 4: Data: descriptive statistics of $\hat{Z}_1$

5.1 Unit Root and Stationary Test

To explore the relation between ODAX and the stock options of the DAX components, the $\hat{Z}_1$ is used to represent the real data. For $\hat{Z}_1$ can be considered as a VAR(p) process, the cointegration test is to be applied for this dynamic series.

The idea of the cointegration is that, if the linear combinations of non-stationary serieses are stationary, then, the variables are said to be cointegrated. (referring to Maddala and Kim (1998)). The unit root tests are applied at first to test the stationary of the serieses.

options	ODAX	DAIMLER Chrysler	SIEMENS	ALLIANZ
ADF Test	-2.4901**	-3.7001	-1.95555**	-2.6254*
p-value	0.31	0.00	0.72	0.00
options	DEUTSCHE BANK	SAP	MÜNCHENER RE	Deutsche Tel
ADF Test	-2.295**	-4.6145	-2.8338**	-3.8226
p-value	0.00	0.03	0.00	0.36
significant level	1% -3.43	5% -2.86	10% -2.57	

Table 5: ADF: test result

Table 5 is the result of Augmented Dickey-Fuller (ADF) test. The H_0 of the ADF test is that the series $\hat{Z}_{t,1}$ has a unit root and is nonstationary. In the table, the test results of the $\hat{Z}_{t,1}$ for ODAX, Siemens, Deutsche Bank and Münchener Rück are in the range of the 10% significant level. Allianz is within 5% significant level. These are all nonstationary serieses, whereas the p-value for the serieses of Allianz, Deutsche Bank and Münchener Rück are quite low. According to the ADF test, the $\hat{Z}_{t,1}$ serieses for Daimler Chrysler, SAP and Deutsche Telekom are stationary.

In order to prove the result of the ADF test, another unit root test, KPSS test, is applied. In order to use the KPSS test, the optimal lag order p has to be firstly determined. We take the lag order suggested by Akaike Information Criterion (AIC). The test result is in Table 6. The H_0 of the KPSS test is then, that the series is stationary and against a unit root. As it is shown in Table 6, the serieses for ODAX and 6 stock options reject the null hypothesis, which means, that they are nonstationary and each of them has a unit root. Only the series for Deutsche Telekom doesn't reject the null hypothesis. Therefore, both unit root tests indicate, that the $\hat{Z}_{t,1}$ for Deutsche Telekom is stationary. As a result, the $\hat{Z}_{t,1}$ series for Deutsche Telekom cannot be considered in further cointegration test. For security, we take the ODAX and other 6 stock options into further joint analysis.

options	ODAX	DAIMLER Chrysler	SIEMENS	ALLIANZ
optimal p	3	2	4	2
KPSS test	0.6696**	0.3864**	0.671**	0.5305**
options	DEUTSCHE BANK	SAP	MÜNCHENER RE	Deutsche Tel
optimal p	2	2	2	8
KPSS test	0.5304**	0.6197**	0.6178**	0.0636
significant level	1% 0.119	5% 0.146	10% 0.216	

Table 6: KPSS: test result

5.2 The Cointegration Test

Figure 11 displays the $\hat{Z}_{t,1}$ of all the 7 serieses, only without the series of Deutsche Telekom. To explore the cointegration among the serieses, the result of the Johansen Trace test shows in Table 7, which suggests, that all of these serieses are cointegrated. That means, they have long-run relationships with each other.

r0	LR	p-val	90%	95%	99%
0	226.43	0	129.22	134.54	144.91
1	152.38	0	98.98	103.68	112.88
2	103.15	0.0001	72.74	76.81	84.84
3	71.01	0.0006	50.5	53.94	60.81
4	42.03	0.0068	32.25	35.07	40.78
5	21.57	0.031	17.98	20.16	24.69
6	4.28	0.3843	7.6	9.14	12.53

Table 7: Johansen Trace Test for: ODAX and other 6 components

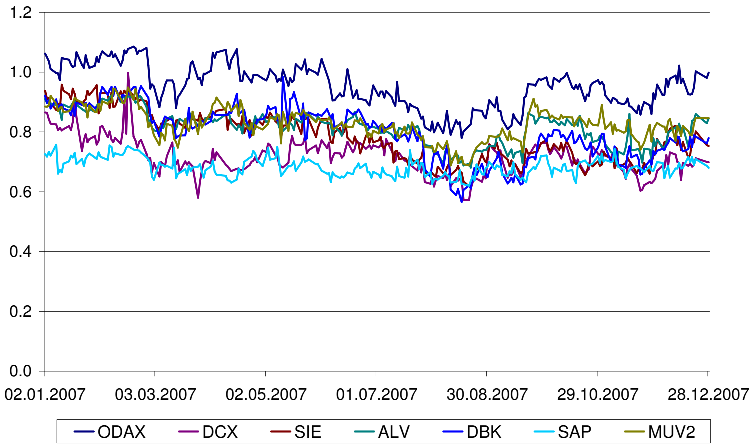

Figure 11: Time series of $\hat{Z}_1$

In order to get more detailed information, the DAX components are divided into 3 groups.

The First group is for the industry companies: Daimler Chrysler and Siemens. As shown in Figure 12 and the result of Johansen trace test in Table 8, the three serieses have similar tendence, especially $\hat{Z}_{t,1}$ for ODAX and Simens options. The result of Johansen Trace Test shows all three serieses are cointegrated.

r0	LR	p-val	90%	95%	99%
0	46.96	0.0014	32.25	35.07	40.78
1	24.8	0.0096	17.98	20.16	24.69
2	4.45	0.3607	7.6	9.14	12.53

Table 8: Johansen Trace Test for: ODAX and Daimler Chrysler, Siemens

The second group is for the financial companies: Allianz Deutsche Bank and Münchener Rück. Figure 13 and Table 9 present the result. The four $\hat{Z}_{t,1}$

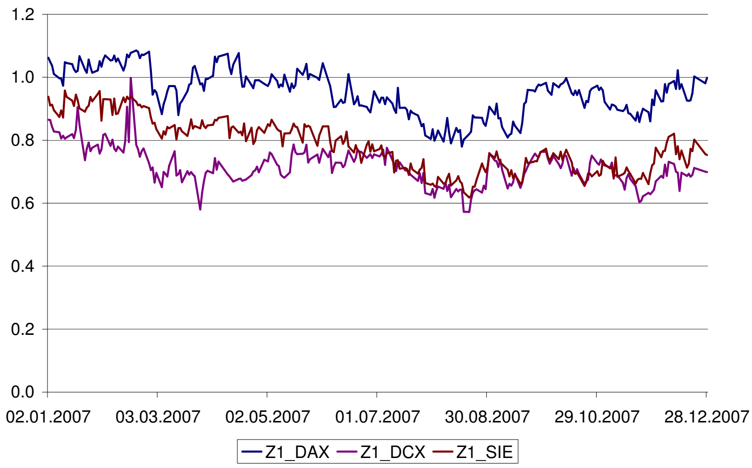

Figure 12: Time series of $\hat{Z}_1$ for ODAX and Daimler Chrysler, Siemens

serieses are also cointegrated. The coefficient of Johansen trace test in rank 3 is within the range of 90% significant level.

r0	LR	p-val	90%	95%	99%
0	112.74	0	50.5	53.94	60.81
1	61.62	0	32.25	35.07	40.78
2	26.26	0.0055	17.98	20.16	24.69
3	6.24	0.1788	7.6	9.14	12.53

Table 9: Johansen Trace Test for: ODAX and Allianz, Deutsche Bank, Münchener Rück

The third group is then for the only information technique company in our list: SAP. The $\hat{Z}_{t,1}$ of SAP is still cointegrated with ODAX. But the coefficient of Johansen trace test in rank 1 is between 90% and 95% significant level. It can be understood, that the cointegration argument is not so strong as that for the last two groups.

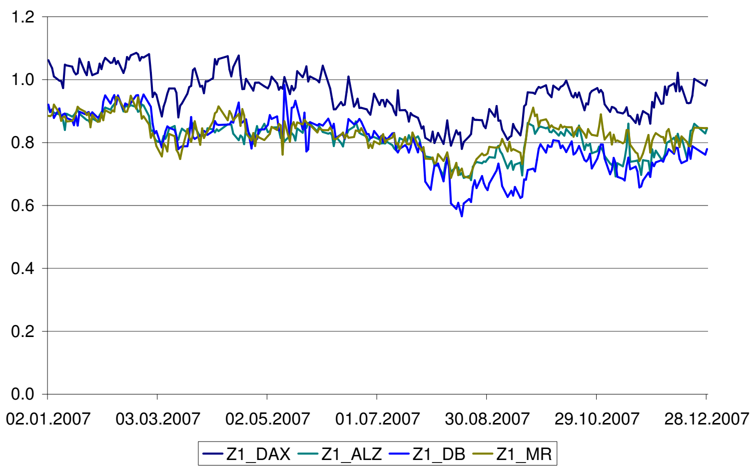

Figure 13: Time series of $\hat{Z}_1$ for ODAX and Allianz, Deutsche Bank, Münchener Rück

r0	LR	p-val	90%	95%	99%
0	43.49	0	17.98	20.16	24.69
1	7.68	0.0967	7.6	9.14	12.53

Table 10: Johansen Trace Test for: ODAX and SAP

As $\hat{Z}_{t,1}$ is the factor loading of $\hat{m}_1$, which represents the (log-)IVS, the relationship among $\hat{Z}_{t,1}$ can be considered as the relationship among these options. The results of the cointegration tests indicate the existence of cointegration between the first factor loadings for DAX index options and the stock options. That means, there exists a long-term relationship between the DAX index options and its stock options. Furthermore, the results of the cointegration also reflect, that there are also long-term relationships among the stock options in the same market.

As the options chosen in this thesis are based on the same market, it can be considered that they are influenced by the same market enviroment, especially when they are in the same local market. In the equity market, the

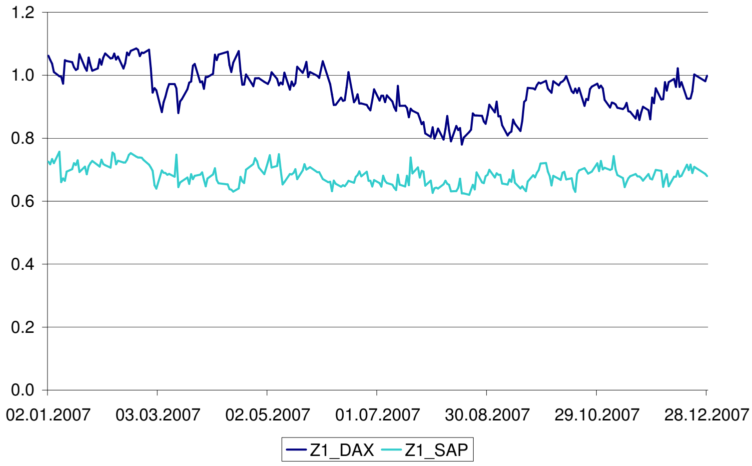

Figure 14: Time series of $\hat{Z}_1$ for DAX and SAP

DAX index is the capital weighted index of 30 major German companies. The underlying prices of DAX index are then determined by these 30 companies. As the liquid components are chosen from the DAX 30 companies, it can be considered that these 7 companies underlying prices have impact on the DAX index underlying prices, that indicates the cointegration among the first factor loadings of the DAX options and stock options. Since the market is arbitrage free, the cointegration among the stock options reflects that they are influenced by the same macro enviroment. Their fluctuations through the time denpend on the fluctuations of global and local markets.

Furthermore, as the liquid components are divided into three different industry groups, the slightly differenciated results from the trace test still suggest, that the factor loadings of industrial and financial companies may be related closer than information technique companies to the factor loadings of ODAX. This may be caused, that the options of the first two groups are traded more actively. At the equity market, the industrial and financial companies in our list may also have larger capital weights than SAP in the DAX index, that means, the first two industry groups may have more influence power on the equity index.

6 Conclusion

The purpose of this master thesis is to explore the relationship between the DAX index options and the stock options, which are the liquid components of DAX index. Since the implied volatility is an approximation of the options' prices and contains multidimentional information of the options, we calculate the implied volatility for all of our observations as a first step. In order to reduce the dimention to make our analysis more observable, the dynamic semiparametric factor models (DSFM) are used, which model the implied volatility surface and produce the nonparametric factor functions and the factor loadings. The first two steps simplifies the third step for the joint analysis, since the factor loadings resulted from DSFM are used to represent the original data. In the third step, the factor loadings are considered as a vector autoregression (VAR) process. Applying the unit root tests and trace test, the cointegration between the factor loadings are proved. Since $\hat{Z}_1$ is the factor loading of $\hat{m}_1$, which is the nonparametric function represents the implied volatility. The cointegration test indicates then, that the implied volatilities of the DAX index options and DAX components stock options have long term relation, which means, that the DAX index options and the stock options also have long term relation.

This thesis still left several theoretical work for further research. As the stock options trading at EUREX are in American style, the Black-Scholes formula doesn't solve the situation, when the options are exercised earlier than expiration. The choice for closing prices for the daily spot prices is also critical. The futures prices calculated from the daily closing prices may also influence the accuracy the result.

Moreover, the long-run relationship between the DAX index optioins and the stock options, moreover, the relationship among the stock options indicates, that the investors cannot disperse the risk by trading with liquid components under one equity index, since the options are influenced by the same market environment. Furthermore, because of the different trading sizes and different capital weights in the index, different industries components may have different impact on the index options.

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